Calculus AB

Lesson: Wednesday, April 8

Learning Target:

Students will integrate exponential equations with any base.

Let's Get Started:

Read Article: Review u-substitution

Watch Video: Integrate Exponential Equations

Practice:

1. The video you just watched derived the following formula: $\int a^x \, dx = \frac{a^x}{\ln a} + C$
2. Now that we have this formula we can use this instead of having to change the base to e for each problem.
3. This formula was used to complete the following problem:

$$\int 3^x \, dx = \frac{1}{\ln 3} \cdot 3^x + C$$

Here is another worked out example:

$$\int \frac{2^x}{2^x+1}\,dx$$

We start off with u-substitution. Let $u = 2^x + 1$. Then, $du = 2^x \cdot \ln 2\,dx$ or $\frac{du}{\ln 2} = 2^x\,dx$. We can substitute these in now.

$$\begin{aligned}
&\int \frac{2^x}{2^x+1}\,dx \\
&= \int \frac{2^x}{u}\,dx \\
&= \int \frac{\frac{du}{\ln 2}}{u} \\
&= \frac{1}{\ln 2}\int \frac{1}{u}\,du \\
&= \frac{1}{\ln 2}\ln|u| + C \\
&= \boxed{\frac{1}{\ln 2}\ln|2^x+1| + C}
\end{aligned}$$

Here are two more worked out examples.

$$\int x\left(2^{-2x^2}\right)dx$$

$$u=-2x^2$$

$$du=-4x\,dx$$

$$dx=-\frac{du}{4x}$$

$$\int x\left(2^{-2x^2}\right)dx=\int \cancel{x}\left(2^u\right)\cdot\left(-\frac{du}{4\cancel{x}}\right)=-\frac{1}{4}\int 2^u\,du=-\frac{2^u}{4\ln 2}+C=-\frac{2^{-2x^2}}{4\ln 2}+C$$

$$\int_0^2\left(3^x-2^x\right)dx$$

$$\int_0^2\left(3^x-2^x\right)dx=\left[\frac{3^x}{\ln 3}-\frac{2^x}{\ln 2}\right]_0^2=\left(\frac{3^2}{\ln 3}-\frac{2^2}{\ln 2}\right)-\left(\frac{3^0}{\ln 3}-\frac{2^0}{\ln 2}\right)=\frac{8}{\ln 3}-\frac{3}{\ln 2}\approx 2.95$$

Practice: Evaluate the following.

Suppose the rate of growth of bacteria in a Petri dish is given by $q(t) = 3^t$, where t is given in hours and $q(t)$ is given in thousands of bacteria per hour. If a culture starts with 10,000 bacteria, find a function $Q(t)$ that gives the number of bacteria in the Petri dish at any time t. How many bacteria are in the dish after 2 hours?

$$\int 2 \cdot 3^x \, dx$$

$$\int 3 \cdot 5^x \, dx$$

Answer Key:

Once you have completed the problem, check your answers here.

Suppose the rate of growth of bacteria in a Petri dish is given by $q(t) = 3^t$, where t is given in hours and $q(t)$ is given in thousands of bacteria per hour. If a culture starts with 10,000 bacteria, find a function $Q(t)$ that gives the number of bacteria in the Petri dish at any time t. How many bacteria are in the dish after 2 hours?

We have

$$Q(t) = \int 3^t\,dt = \frac{3^t}{\ln 3} + C.$$

Then, at $t = 0$ we have $Q(0) = 10 = \frac{1}{\ln 3} + C$, so $C \approx 9.090$ and we get

$$Q(t) = \frac{3^t}{\ln 3} + 9.090.$$

At time $t = 2$, we have

$$Q(2) = \frac{3^2}{\ln 3} + 9.090$$

$$= 17.282.$$

After 2 hours, there are 17,282 bacteria in the dish.

$$\int 2 \cdot 3^x\,dx$$

$$\frac{2 \cdot 3^x}{\ln 3} + C$$

$$\int 3 \cdot 5^x\,dx$$

$$\frac{3 \cdot 5^x}{\ln 5} + C$$

Additional Practice:

[Additional Practice with Answers]

In your Calculus book Read through Section 5.5 and complete problems 64, 66, 68, 70 on page 367